1

Patterns, Sequences, Rules . . .

Analyzing Sequences as Rules

WARM UP

1. List six consecutive numbers.
2. List six consecutive even numbers.
3. List six consecutive multiples of seven.
4. List six consecutive multiples of five that are decreasing.
5. List six consecutive prime numbers.

LEARNING GOALS

- Write sequences of numbers generated from the creation of diagrams and written contexts.
- State varying growth patterns of sequences.

KEY TERMS

- sequence
- term
- ellipsis

You are surrounded by patterns every day, and you have examined many mathematical patterns in school. How are patterns of numbers related to the linear relationships you have studied?

Getting Started

Sequences of Events

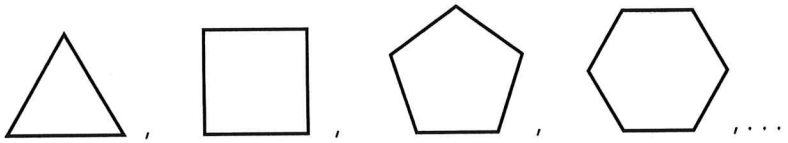
A **sequence** is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. A **term** in a sequence is an individual number, figure, or letter in the sequence.

Here are some example sequences.

Often, only the first few terms of a sequence are listed, followed by an *ellipsis*. An **ellipsis** is a set of three periods, which stands for "and so on."

Sequence A:
2, 4, 6, 8, 10, 12, . . .

Sequence B:



Sequence C:
3, 9, 27, 81, . . .

1. **Identify the next term in each sequence. Explain how you determined each answer.**

A → 14 → counting by two

B → a heptagon - a polygon with 7 sides → each term adds another side

C → 243 → multiplying by 3

2. **Generate a sequence, given this information:**

Starting term: 1
Rule: Multiply each term by 3 and then subtract 1 to get the next term

1, 2, 5, 14, 41 . . .

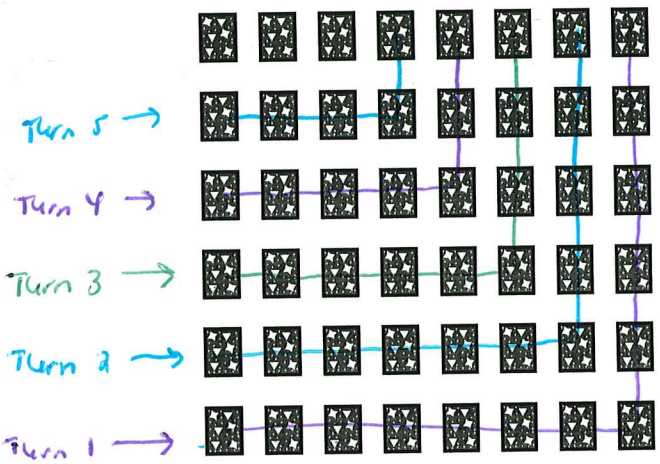
1 × 3 = 3 → 3 − 1 = (2)
2 × 3 = 6 → 6 − 1 = (5)
5 × 3 = 15 → 15 − 1 = (14)
14 × 3 = 42 → 42 − 1 = (41)

ACTIVITY 1.1

Analyzing a Variety of Different Sequences

Taking Apart a Card Trick

Matthew is performing a card trick. It is important that he collect the cards shown in a particular order. Each turn, he collects all of the cards in the right-most column, and all the cards in the bottom row.



1. **Write a sequence to show the number of cards removed during each of the first five turns.**

   13, 11, 9, 7, 5

2. **Write a sequence to show the number of cards remaining after each of the first five turns.**

   Start w/ 48 and subtract 13 → Term 1 = 35 Subtract 11 = 24 Subtract 9 = 15 Subtract 7 = 8 Subtract 5 = 3

   35, 24, 15, 8, 3

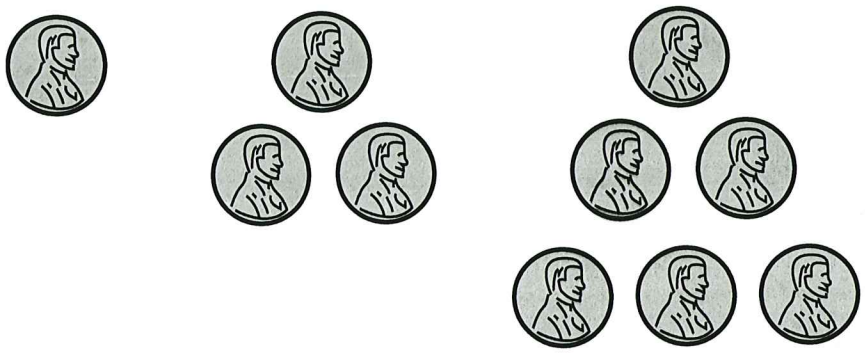
   * Or you could count the # left each time.

3. **What pattern is shown in each sequence?**

   In the first sequence - the # of cards goes down by 2 each time. In the second sequence, the # of cards decreases by the values shown in the first sequence starting with 11.

Arranging Pennies

Lenny is making arrangements with pennies. He has made three penny arrangements, and now he wants to make five more arrangements. Each time he adds another arrangement, he needs to add one more row to the base, and the row needs to have one more penny than the last row in the previous arrangement.



4. **Write the first eight terms in the sequence that represents this situation. Each term should indicate the total number of pennies in each arrangement. Explain your reasoning.**

1, 3, 6, 10, 15, 21, 28, 36

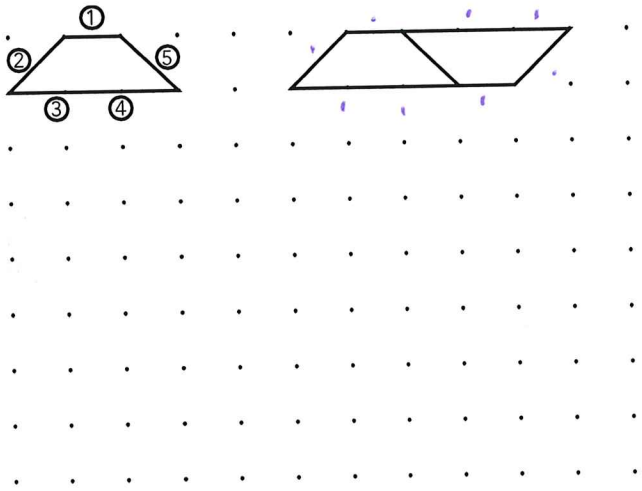
Every increase is one more than the next.
+2, +3, +4, +5...

5. **Explain why the pattern does not increase by the same amount each time.**

Every time a new row is added, the triangle gets wider & adds one more penny than the previous row.

Arranging Classroom Tables

Some schools purchase classroom tables that have trapezoid-shaped tops rather than rectangular tops. The tables fit together nicely to arrange the classroom in a variety of ways. The number of students that can fit around a table is shown in the first diagram. The second diagram shows how the tables can be joined at the sides to make one longer table.



6. **Write the first five terms in the sequence that represents this situation. Each term should indicate the total number of students that can sit around one, two, three, four, and five tables. Assume that the tables are joined at the sides, as shown in the second diagram above. Explain your reasoning.**

5, 8, 11, 14, 17

Each time a table is added it takes one seat away but adds 4 more.

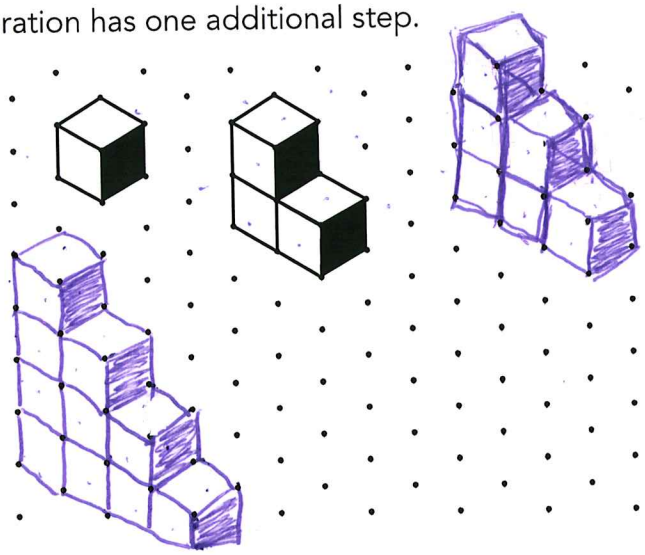
7. **The first trapezoid table seats five students. Explain why each additional table does not have seats for five students.**

When they touch - one seat is covered up.

A configuration is another way of saying an arrangement of things.

Building Stairs

Dawson is stacking cubes in configurations that look like stairs. Each new configuration has one additional step.



8. **Write the first five terms in the sequence that represents this situation. Each term should indicate the number of faces shown from the cubes in the configuration. The bottom faces are not shown. The first cube has five shown faces. Explain your reasoning. Show your work.**

5, 12, 21, 32, 45

The # of faces increased by 7, then 9, then 11, then 13 — 2 more each time. The way they are stacked covers 2 faces on the one block but only one on the other.

9. **Predict the number of shown faces in a stair configuration that is seven cubes high. Show your work.**

45+15=60 60+17=77

Babysitting

Every Friday, Sarah earns $14 for babysitting. Every Saturday, Sarah spends $10 going out with her friends.

10. **Write a sequence to show the amounts of money Sarah has every Friday after babysitting and every Saturday after going out with her friends for five consecutive weeks. The sequence should have 10 terms. Explain your reasoning.**

14, 4, 18, 8, 22, 12, 26, 16, 30, 20

The pattern begins with 14 & alternates subtracting 10 and adding 14.

Recycling

The first week of school, Ms. Sinopoli asked her class to begin collecting cans for recycling. The students started bringing in cans the second week of school. They collected 120 cans per week.

11. **Write a sequence to show the running total number of cans collected through the first nine weeks of school. Explain your reasoning.**

0, 120, 240, 360, 480, 600, 720, 840 960

Every term is calculated by adding 120 to the previous one. You could also multiply by 120.

NOTES

TALK the TALK

Looking Back

There are many different patterns that can generate a sequence. Some possible patterns are:

- adding or subtracting the same number each time,
- multiplying or dividing by the same number each time,
- adding a different number each time, with the numbers being part of a pattern, and
- alternating between adding and subtracting.

The next term in a sequence is calculated by determining the pattern of the sequence and then using that pattern on the last known term of the sequence.

1. **Look back at the sequences you analyzed in this lesson. Describe the pattern of each sequence by completing the table shown.**

Sequence Name	Increases or Decreases	Describe the Pattern
Taking Apart a Card Trick		
Arranging Pennies		
Arranging Classroom Tables		
Building Stairs		
Babysitting		
Recycling		

2. **Which sequences are similar? Explain your reasoning.**

Assignment

Write

Define each term in your own words.

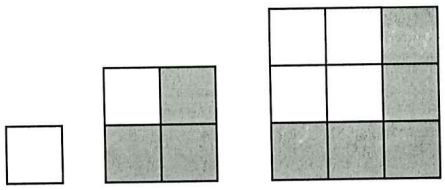
1. sequence
2. term
3. ellipsis

Remember

There are many different patterns that can generate a sequence. The next term in a sequence is calculated by determining the pattern of the sequence and then using that pattern on the last known term of the sequence.

Practice

1. Amanda is training to run a marathon. She must follow a strict schedule to make sure she is ready for the race. She will start her training by running two miles the first week. She wants to run one fewer mile the next week, and then three more miles the week after that. She will continue this pattern during her entire training regimen.
   a. Write a sequence for the number of miles that Amanda will run the first 10 weeks of her training. Explain your reasoning.
   b. In which week of training will Amanda run seven miles?
   c. Amanda needs to run 26 miles in the final week of her training. In which week will Amanda reach her goal? Explain your answer.
   d. Amanda is considering changing her regimen by running two miles the first week and then running an additional two miles each subsequent week. Write a sequence for the number of miles that Amanda would run the first 10 weeks of her training if she followed the new regimen. Explain your reasoning.
   e. In which week would Amanda reach her goal of 26 miles, if she followed the new regimen? Explain your reasoning.
2. Amanda chooses to continue with the first training regimen. Because it will take a long time to train, Amanda decides that during her periods of rest, she will sew a quilt to have as a remembrance of her achievement. She will add squares to the quilt every two weeks using the pattern shown (added squares are shaded).



Weeks 1 and 2 | Weeks 3 and 4 | Weeks 5 and 6

   a. Write a sequence for the first 10 terms generated by this situation. Each term should represent the number of squares that the quilt will have. The first term has one square. Explain your reasoning.
   b. By the end of her training regimen, how many squares will the quilt have? Explain your reasoning.

Stretch

Consider the sequence 6, 11, 16, 21 . . . . What is the 50th term in this sequence?

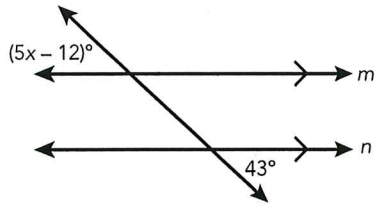
Review

Sketch a graph of each equation. Identify the slope and *y*-intercept.

1. $y = 4x - 1$
2. $y = 4x$
3. $y = x + 4$
4. $y = x - 4$

Solve for *x*.

5.



6.